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Time and Carrier Frequency offset in OFDM Systems using Joint Maximum Likelihood Estimation without additional pilots

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Abstract

In this paper, we present and evaluate the joint maximum likelihood (ML) estimation of the time and carrier-frequency offset in orthogonal frequency division multiplexing (OFDM) systems. The main element is the OFDM, it contains the sufficient information to perform synchronization. Redundant information contained within the cyclic prefix enables this estimation without additional pilots. Simulations show that the frequency estimator may be used in a tracking mode and the time estimator in an acquisition mode. Our novel algorithm exploits the cyclic prefix preceding the OFDM symbols, thus reducing the need for pilots.

1 Introduction

Orthogonal frequency-division multiplexing (OFDM) systems have recently gained increased interest. OFDM is used in the European digital broadcast radio system and is being investigated for other wireless applications such as digital broadcast television and mobile communication systems, as well as for broadband digital communication on existing copper networks. We address two problems in the design of OFDM receivers. One problem is the unknown OFDM symbol arrival time. Sensit-ivity to a time offset is higher in multicarrier systems than in single-carrier systems and has been discussed in [3] and [4]. A second problem is the mismatch of the oscillators in the transmitter and receiver. The demodulation of a signal with an offset in the carrier frequency can cause a high bit error rate and may degrade the performance of a symbol synchronizer. A symbol clock and a frequency offset estimate may be generated at the receiver with the aid of pilot symbols known to the receiver [6,7], by maximizing the average log-likelihood function. Redundancy in the transmitted OFDM signal also offers the opportunity for synchronization. We present and evaluate the joint maximum likelihood (ML) estimation of the time and carrier-frequency offset in OFDM systems. The key element that will rule the discussion is that the OFDM data symbols already contain sufficient information to perform synchronization. Our novel algorithm exploits the cyclic prefix preceding the OFDM symbols, thus reducing the need for pilots.

2 The OFDM system model

The baseband, discrete-time OFDM system model as shown in fig.1. The complex data symbols are modulated by means of an inverse discrete Fourier transform (IDFT) on N parallel subcarriers. The resulting OFDM symbol is serially transmitted over a discrete-time channel, whose impulse response we assume is shorter than L samples. At the receiver, the data are retrieved by means of a discrete Fourier transform (DFT). An accepted means of avoiding inter-symbol interference (ISI) and preserving orthogonality between subcarriers is to copy the last L samples of the body of the OFDM symbol (N samples long) and append them as a preamble | the cyclic prefix | to form the complete OFDM symbol [1,2]. The effective length of the OFDM symbol as transmitted is this cyclic prefix plus the body ($L + N$ samples long). The insertion of a cyclic prefix can be shown to result in an

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equivalent parallel orthogonal channel structure that allows for simple channel estimation and equalization. In the following analysis we assume that the channel is non-dispersive and that the transmitted signal *s*(*k*) is only affected by complex additive white Gaussian noise (AWGN) *n*(*k*). We will, however, evaluate our estimator's performance for both the AWGN channel and a time-dispersive channel.

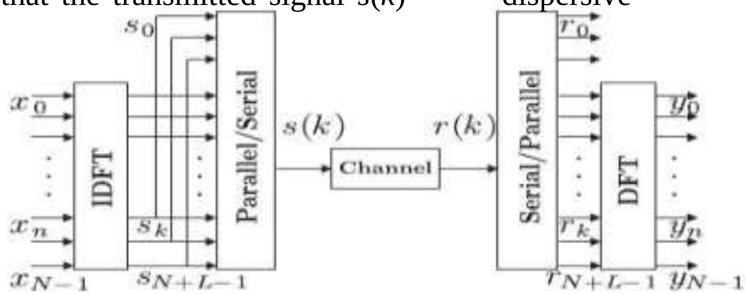

Fig. 1: The OFDM system, transmitting subsequent blocks of N complex data

Consider two uncertainties in the receiver of this OFDM symbol: the uncertainty in the arrival time of the OFDM symbol and the uncertainty in carrier frequency. The first un-certainty is modeled as a delay in the channel impulse response δ(*k* - *θ*), where *θ* is the integer valued unknown arrival time of a symbol. The latter is modeled as a complex multiplicative distortion of the received data in the time domain $e^{j2\pi\varepsilon k/N}$, where *ε* denotes the difference in the transmitter and receiver oscillators as a fraction of the inter-carrier spacing (1=*N* in normalized frequency). Notice that all subcarriers experience the same shift ε. These two uncertainties and the AWGN thus yield the received signal

$$r(k) = s(k - \theta)e^{j2\pi\varepsilon k/N} + n(k)$$
----(1)

Two other synchronization parameters are not accounted for in this model. First, an offset in the carrier phase may affect the symbol error rate in coherent modulation. If the data is differentially encoded, however, this effect is eliminated. An offset in the sampling frequency will also affect the system performance. We assume that such an offset is negligible. Now consider the transmitted signal *s*(*k*). This is the DFT of the data symbols x_k, which we assume are independent. Hence, *s*(*k*) is a linear combination of independent, identically distributed random variables. If the number of subcarriers is sufficiently large, we know from the central limit theorem that *s*(*k*) approximates a complex Gaussian process whose real and imaginary parts are independent. This process, however, is not white, since the appearance of a cyclic prefix yields a correlation between some pairs of samples that are spaced *N* samples apart. Hence, *r*(*k*) is not a white process, either, but because of its probabilistic structure, it contains information about the time offset ε and carrier frequency offset ε. This is the crucial observ-ation that offers the opportunity for joint estimation of these parameters based on *r*(*k*).

3 ML estimation

Assume that 2*N*+*L* consecutive samples of *r*(*k*) (shown in Fig 2), and that these samples contain one complete (*N* + *L*) sample OFDM symbol. The position of this symbol within the observed block of samples, however, is unknown because the channel delay *μ* is unknown to the receiver. Define the index sets

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I = {θ, …….,θ+L-1} and
I^1 = {θ+N, …….,θ+N+L-1} -------------------------(2)

The set I^1 thus contains the indices of the data samples that are copied into the cyclic prefix, and the set I contains the indices of this prefix. Collect the observed samples in the (2*N* +*L*)X1 -vector **r** = [*r*(1)….. *r*(2*N* +*L*)]^T . Notice that the samples in the cyclic prefix and their copies, *r*(*k*), K ε I U I^1 are pairwise correlated, *i.e.*,

∀k ε I : *E*{*r*(*k*)*r**(*k* + *m*)} = {$\sigma s^2 + \sigma n^2$ m=0,
$\sigma s e^{-j2\pi\varepsilon}$ m=N,
Otherwise --------------------(3)

while the remaining samples *r*(*k*)*;* K ε I U I^1 are mutually uncorrelated.

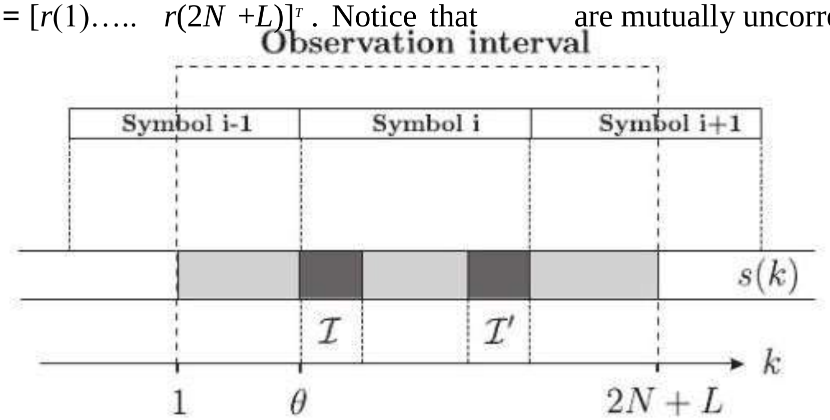

Figure 2: Structure of OFDM signal with cyclicly extended symbols, s(k). The set I contains the cyclic prefix, i.e. the copies of the L data samples in I .

The log-likelihood function for θ and ε, ʌ(θ, ε) is the logarithm of the probability density function *f* (r/θ, ε) of the 2*N* + *L* observed samples in **r** given the arrival time θ and the carrier frequency offset ε. In the following, we will drop all additive and positive multiplicative constants that show up in the expression of the log-likelihood function, since they do not affect the maximizing argument. Moreover, we drop the conditioning on (θ, ε) for notational clarity. Using the correlation properties of the observations **r**, the log-likelihood function can be written as

ʌ(θ, ε) = log f(r/θ, ε) ----------------------- (4)

The ML estimation of θ and ε is the argument maximizing ʌ(θ, ε), we may omit this factor. Under the assumption that **r** is a jointly Gaussian vector, (4) is shown in the Appendix to be

ʌ(θ, ε) = |γ(θ)| cos(2πε+L γ(θ)) - ρφ(θ) ------------------------(5)

where L represent the argument of a complex number.

The first term in (5) is the weighted magnitude of γ(θ) , a sum of *L* consecutive correlations between pairs of samples spaced *N* samples apart. The weighting factor depends on the frequency offset. The term φ(θ) is an energy term, independent of the frequency offset ε . Notice that its contribution depends on the SNR.

The maximization of the log-likeluihood function can be performed in two steps:

Max ʌ(θ, ε) = max max ʌ(θ, ε) = max ʌ(θ, ε_{ML}(θ)) ------------6

The maximum with respect to the frequency offset ε is obtained when the cosine term in (5) equals one. This yields the ML estimation of ε,

ε_{ML}(θ) = -1/2π L γ(θ) + n ---------------------7

where *n* is an integer. A similar frequency offset estimator has been derived in [11] under different assumptions. Notice that by the peri-

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odicity of the cosine function, several maxima are found. We assume that an acquisition, or rough estimate, of the frequency offset has been performed and that $|\varepsilon| < 1/2$; thus $n = 0$. Since $\cos(2\pi\, \varepsilon_{ML}(\theta) + L\, \gamma(\theta)) = 1$, the log-likelihood function of θ (which is the compressed log-likelihood function with respect to ε) becomes

$\Lambda(\theta, \varepsilon_{ML}(\theta)) = |\gamma(\theta)) - \rho\varphi(\theta)|$ ------------------8

and the joint ML estimation of θ and ε becomes

$\theta_{ML} = \arg\max\{|\gamma(\theta)| - \rho\varphi(\theta)\}$ ----------------9

$\varepsilon_{ML} = -1/2\pi\, L\, \gamma(\theta_{ML})$ ---------------------10

Notice that only two quantities affect the log-likelihood function : the number of samples in the cyclic prefix *L* and the correlation coefficient given by the SNR. The former is known at the receiver, and the latter can be fixed. Basically, the quantity $\gamma(\theta)$ provides the estimates of θ and ε . Its magnitude, compensated by an energy term, peaks at time instant θ_{ML}, while its phase at this time instant is proportional to ε_{ML}. If ε is *a priori* known to be zero, the log-likelihood function for θ becomes $\Lambda(\theta) = \mathrm{Re}\{\gamma(\theta)\} - \rho\varphi(\theta)$ and θ_{ML} is its maximizing argument. This estimator and a low-complexity variant are analyzed in [9].

In an OFDM receiver, the quantity $\gamma(\theta)$, which is defined in (6), is calculated online, *cf*. Fig 3. The signals $\Lambda(\theta, \varepsilon_{ML}(\theta))$ and $-(1/2)L\, \gamma(\theta)\}$ (whose values at the time instants θ_{ML} yield the frequency estimates) are shown in Figure 4. Notice that (12) and (13) describe an open-loop structure. Closed-loop implementations based on (5) and (11) may also be considered. In such structures the signal $\Lambda(\theta, \varepsilon_{ML}(\theta))$ is typically fed back in a phase-locked loop (PLL). If we can assume that θ is constant over a certain period, the integration in the PLL can significantly improve the performance of the estimators.

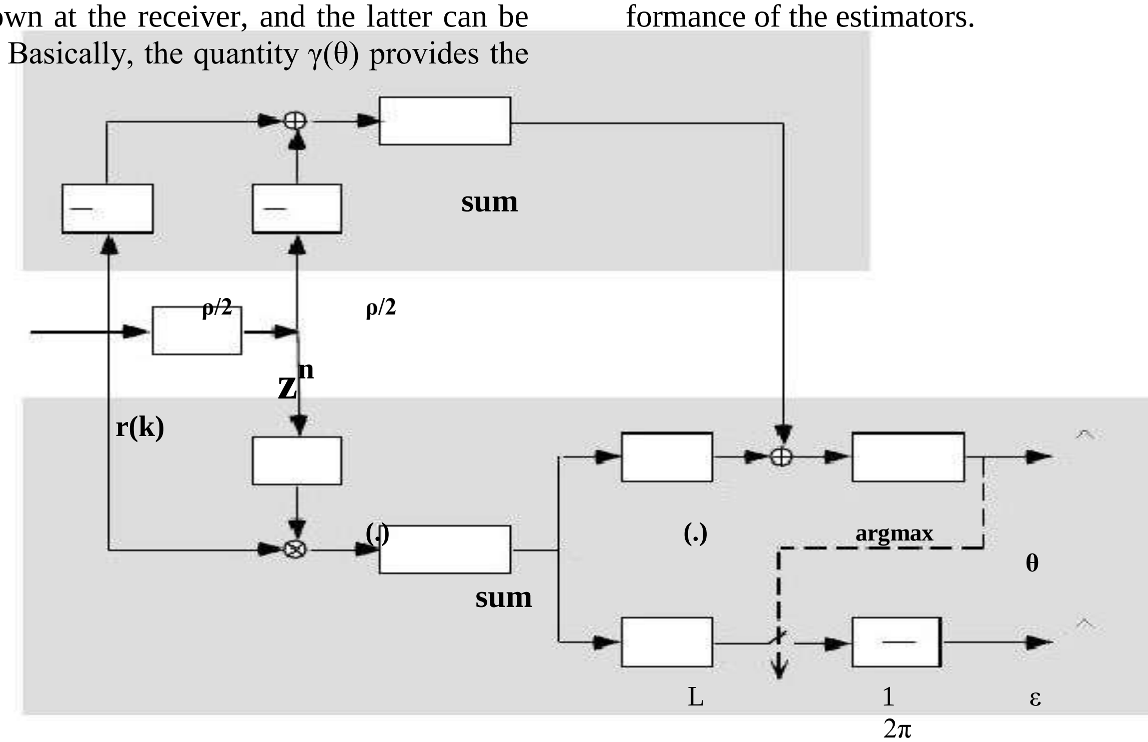

Figure 3: Structure of the estimator

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4. Results

Performance results for the AWGN channel are shown in Figures 5 and 6. First, the estimator mean-squared error as a function of L is estimated. Figure 5 shows the estimator performance for SNR values of 4 dB, 10 dB and 16 dB. Notice that the performance of the time estimator is asymptotically independent of L, provided that the cyclic prefix is longer than a certain threshold value. This threshold value decreases with the SNR. Both the time estimator and the frequency estimator exhibit such a performance threshold based on L. However, notice that as L increases beyond these respective thresholds, only the frequency estimator will show continued improvement. Thus, for the AWGN channel and from a time synchronization viewpoint, there is very little advantage in increasing the length of the cyclic prefix beyond the time estimator's threshold. Second, the estimator variances as a function of SNR for L = 4, L = 8, and L = 15 are shown in Figure 6. Notice that even in these plots a threshold phenomenon as in Figure 5 occurs.

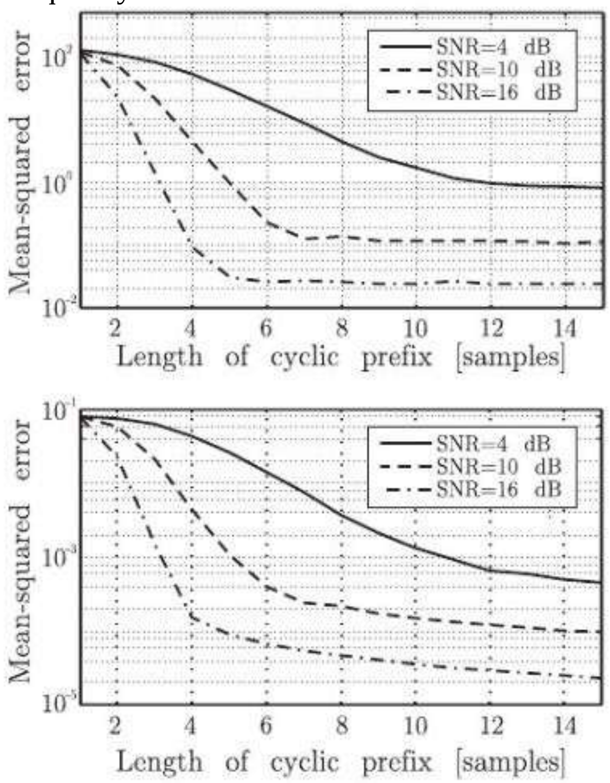

Fig. 5: Performance of the time (top) and frequency (bottom) estimators for the AWGN channel (4 dB, 10 dB, and 16 dB).

The above results do not directly apply to a time dispersive channel environment. Therefore, we also consider the performance of our estimators in a wireless system operating at 2 GHz with a bandwidth of 5 MHz. An outdoor dispersive, fading environment with micro-cell characteristics is chosen: the channel has an exponentially decaying power delay profile with root mean squared width equal to 0:4 μs and a

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maximum delay spread of 3 μs (corresponding to 15 samples). It is modeled to consist of 15 independent Rayleigh-fading taps [16] and additive noise. We choose a cyclic prefix consisting of 15 samples. This choice avoids ISI, while the loss of power and bandwidth due to the cyclic prefix (L=(N+L)) is about 5%. This system transmits about 18,000 OFDM symbols per second, each containing 256 complex information symbols. In this dispersive environment the definition of θ is ambigious. We define the true delay as the center-of-gravity of the channel impulse response. Moreover, we define the SNR as SNR = $\sigma_s^2 P_h / \sigma_n^2$, where P_h is the sum of the average power in all channel taps.

The error floor in Figure 6 clearly shows the performance degradation caused by the dispersive channel as compared with the corresponding curves for the AWGN channel. In the dispersive case, the estimators operate in an environment for which they are not designed (they are not optimal). Signals passed through the AWGN channel will have the simple, pairwise correlation structure (3), but signals passed through a dispersive channel generally have a more complex correlation structure. Depending on the application and the presence of a high performance channel estimator/equalizer, the performance of the time estimate in Figure 6 (standard deviation of 1-2 samples) may be good enough to generate a stable clock. In most situations this performance will suffice at least in an acquisition mode. The frequency offset estimator shows an error standard deviation of less than 2% of the inter-tone spacing (see Figure 6).

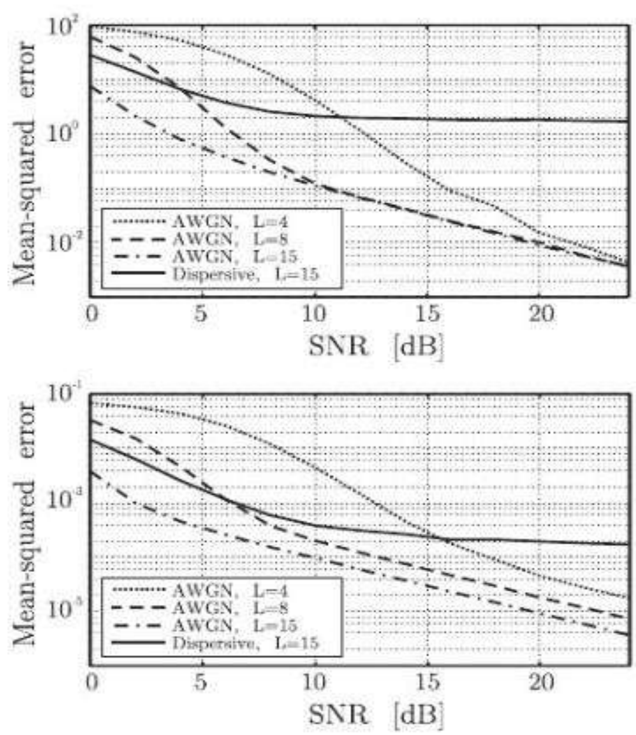

Fig 6: Performance of the time (top) and frequency (bottom) estimators for the AWGN channel (L = 4, L = 8, and L = 15) and the dispersive channel (L = 15).

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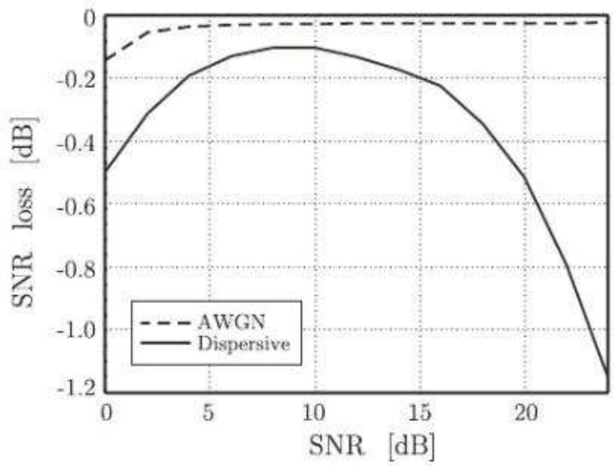

Fig 7: Performance of the frequency estimator for the AWGN channel and the dispersive channel (L = 15). The number of subcarriers is N = 256.

Finally, the performance of the frequency estimator is plotted in Fig.7. The SNR loss is a function of ε. We assume that a frequency offset can be corrected using the estimate ε_{ML} and we thus use the standard deviation of the estimate as the argument in (2). The SNR loss is plotted for the AWGN channel and the dispersive channel. Notice that even for the dispersive channel this loss does not exceed 0.5 dB for SNR values between 0 dB and 20 dB.

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