

## FEM USE FOR EVALUATION OF TRANSMISSION ERROR IN GEAR

<sup>1</sup>Shalini rai, <sup>2</sup>Arun Rai

<sup>1,2</sup>Assistant professor in Hindu college of engineering  
rai.shalin@gmail.com, arunrai08@gmail.com

### Abstract

*Gears analyses in the past were performed using analytical methods, which required a number of assumptions and simplifications. In general, gear analyses are multidisciplinary, including calculations related to the tooth stresses and to tribological failures such as like wear or scoring. In this paper, static contact and bending stress analyses were performed, while trying to design spur gears to resist bending failure and pitting of the teeth, as both affect transmission error. This paper is to develop a model to study and predict the transmission error model including the contact stresses, and the torsional mesh stiffness of gears in mesh using the ANSYS software package based on numerical method. The aim is to reduce the amount of transmission error in the gears, and thereby reduce the amount of noise generate.*

**Keywords:** *Torsional mesh stiffness, Transmission error, pitting & Scoring.*

### Introduction and definition of Transmission Error

The static transmission error is defined by Smith. The term transmission error is used to describe the difference between the theoretical and actual relative angular rotations between a pinion and a gear. Its characteristics depend on the instantaneous positions of the meshing tooth pairs. Under load at low speeds (static transmission error) these situations result from tooth deflections and manufacturing errors. In service, the transmission error is mainly caused by:

- Tooth geometry errors: including profile, spacing and run out errors from the manufacturing process;
- Elastic deformation: local contact deformation from each meshing tooth pair and the deflections of teeth because of bending and shearing due to the transmitted load;
- Imperfect mounting: geometric errors in alignment, which may be

introduced by static and dynamic elastic deflections in the supporting bearings and shafts.

The first two types of transmission errors are commonly referred to in the literature. The first case has manufacturing errors such as profile inaccuracies, spacing errors, and gear tooth run out. When the gears are unloaded, a pinion and gear have zero transmission error if there is no manufacturing error. The second case is loaded transmission error, which is similar in principle to the manufactured transmission error but takes into account tooth bending deflection, and shearing displacement and contact deformation due to load. One of the most important criteria for each model was that the potential contact nodes of both surfaces would be created on the nodes near the

intersection point between the pressure line and the involutes curve for that particular tooth. At each particular meshing position, after running ANSYS the results for angular rotation of the gear due to tooth bending, shearing and contact displacement were calculated. In the pinion reference frame: the local cylindrical system number 12 was created by definition in ANSYS. By constraining the all nodes on the pinion in

Where  $Z$  is the gear ratio and  $\theta_p, \theta_g$  is the angular rotation of the input and output gears in radians respectively. The gear was restrained with degrees of freedom in radius and rotating  $\theta_p$  with the pinion having the torque input load and the resulting angular motion of  $\theta_g$ . After compensating for torque and Rotation of the pinion was computed. In this second case  $\theta_g = 0$  and the TE will be positive for forward angular rotation for the particular gear ratio, the results from these two models should be the same, and so the mean of these two angular rotations would give the best estimate of the true static

### The Combined Torsional Mesh Stiffness

Because the number of the teeth in mesh varies with time, the combined torsional mesh stiffness varies periodically. When a gear with perfect involutes profiles is loaded the combined torsional mesh stiffness of the gear causes variations in angular rotation of the gear body. The gear transmission error is related directly to the deviation of the angular rotation of the two gear bodies and the relative angular rotation of the two gears is inversely proportional to the combined torsional mesh stiffness, which can be seen from the results of ANSYS later in this document. The combined torsional mesh stiffness is different throughout the period of

radius and rotating  $\theta_g$  with the gear having a torque input load the model was built. In this case,  $\theta_p = 0$  and  $\theta_g$  is in the opposite direction to that resulting from forward motion of  $\theta_g$  changing the TE result to positive as seen by equation(1).

$$TE = \theta_g - (Z) \theta_p \quad (1)$$

transmission error of the involutes profile gears under load. With the pinion having the torque input load and the resulting angular rotation of the pinion was computed. In this second case  $\theta_g = 0$  and the TE will be positive for forward motion of  $\theta_g$ . After compensating for torque and angular rotation for the particular gear ratio, the results from these two models should be the same, and so the mean of these two angular rotations would give the best estimate of the true static transmission error of the involutes profile gears under load.

meshing position. It decreases and increases dramatically as the meshing of the teeth change from the double pair to single pair of teeth in contact. The combined torsional mesh stiffness of gears is time dependent during involutes action due to the change in the number of contact tooth pairs. Considering the combined torsional mesh stiffness for a single tooth pair contact zone, the single tooth torsional mesh stiffness of a single tooth pair in contact is defined as the ratio between the torsional mesh load ( $T$ ) and the elastic angular rotation ( $\theta$ ) of the gear body. In the single tooth pair contact zone, as the pinion rotates, the single tooth torsional mesh stiffness of the pinion,  $K_P$  is

decreasing while the single tooth torsional stiffness of the gear,  $K_g$ , is increasing. When the pinion rotates to the pitch point P, the single tooth torsional stiffness of both gears is equal because both of them were assumed

$$K_p^B = \frac{T_p^B}{\theta_p^B}$$

$$K_g^B = \frac{T_g^B}{\theta_g^B}$$

$K_p^B$  and  $K_g^B$  are the single tooth torsional mesh stiffness of the single tooth pairs at B of the pinion and gear respectively. The torsional mesh stiffness can be related to the contact stiffness by considering the normal contact force operating along the line of action tangential to the base circles of the gears in mesh. The torsional mesh stiffness can be seen to be the ratio between the torque and the angular deflection. By considering the total normal contact force F, acting along the line of action, the torque T will be given by the force multiplied by the perpendicular distance (base circle radius rb)  $T = F \cdot rb$  if there is one pair gear on contact. The elastic angle of rotation  $\theta$  of the gear body can then be calculated from related to the arc length c, by the base circle radius as  $\theta = c/r$ . The torsional mesh stiffness can then be given by:

$$K_m = \frac{T}{\theta} = \frac{Frb}{\frac{c}{rb}} = \frac{Frb^2}{c}$$

The tooth contact stiffness  $K_{mb}$ , can be seen to be the ratio of the normal contact force F to the displacement along the line of action, which gives  $K_{mb} = F/a$ , where the length a is equal to the arc c length for a small angles  $\theta$ . The relationship between the linear contact stiffness and torsional mesh stiffness then becomes,

$$K_{mb} = \frac{K_m}{r^2b}$$

The contact between the gears is a nonlinear problem. This cannot be put in the form of a linear differential equation if the problem is solved by the equations so here ANSYS was used to study this problem. In this chapter the program ANSYS 10.0 was used to help to solve this nonlinear problem. The gears

were modeled using quadratic two dimensional elements and the contact effect was modeled using 2D surface-to-surface (line-to-line) general contact elements that can include elastic Coulomb frictional effects.

**Table 2.2 Gear Parameters Used in the Model**

Gear Type	Standard involutes, Full-Depth Teeth
Modulus of Elasticity, E	200GPa
Module (M)	3.75 mm
Number of Teeth	27
Pressure Angle	20
Addendum, Dedendum	1.00*M, 1.25*M

### Analysis of Load Sharing Ratio

Under normal operating conditions, the main source of vibration excitation is from the periodic changes in tooth stiffness due to non-uniform load distributions from the double to single contact zone and then from the single to double contact zone in each meshing cycle of the mating teeth. This indicates that the variation in mesh stiffness can produce considerable vibration and dynamic loading of gears with teeth, in

mesh. For the spur involute teeth gears, the load was transmitted between just one to two pairs of teeth gears alternately. The torsional stiffness of two spur gears in mesh varied within the meshing cycle as the number of teeth in mesh changed from two to one pair of teeth in contact. Usually the torsional stiffness increased as the meshing of the teeth changed.

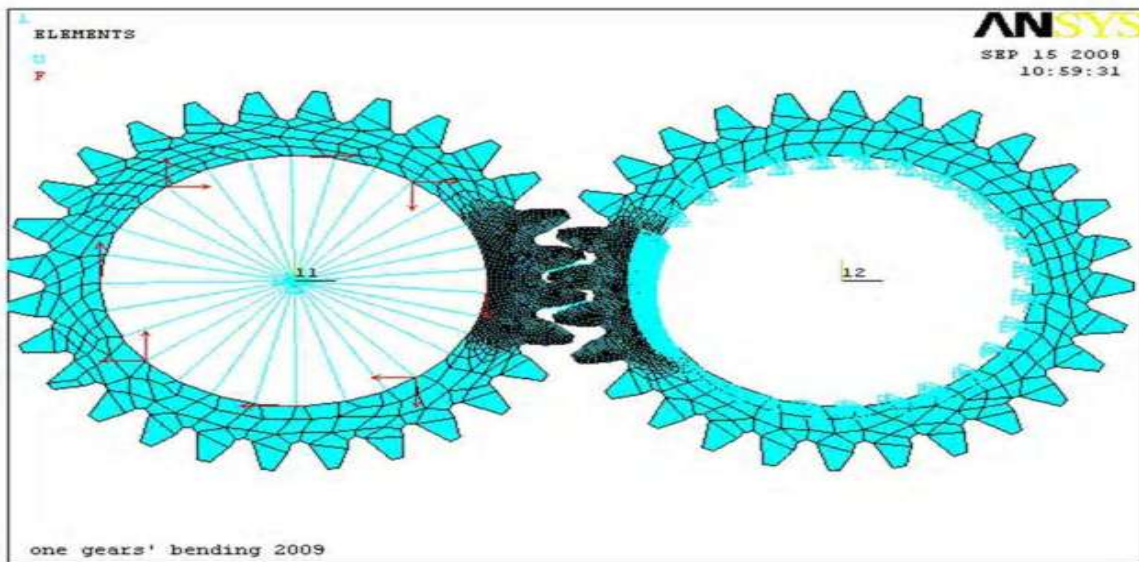


Figure 3-19 Contact elements between the two contact surfaces

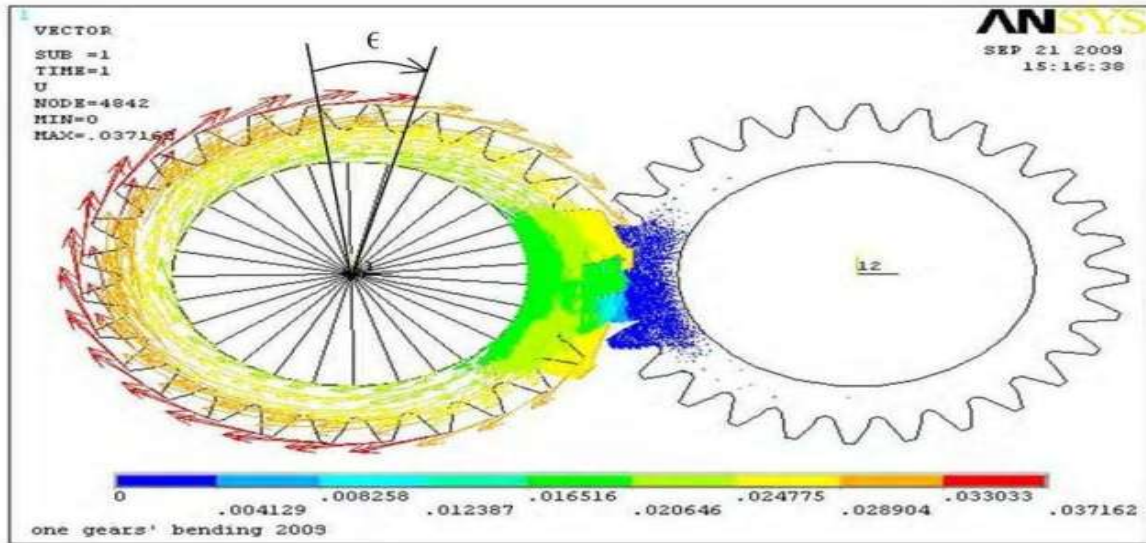


Figure 3.17 Vectors of displacement

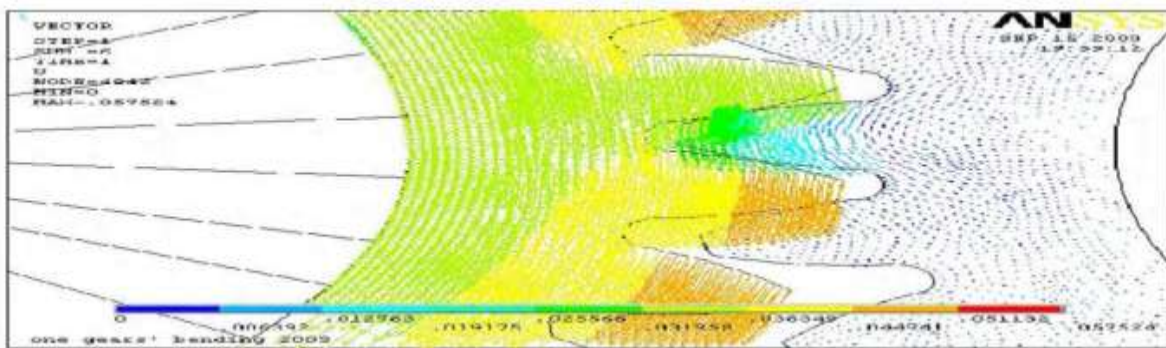


Figure 3-18 Vectors of displacement near the contact surfaces

### 2D FEA Transmission Error Model

In this section to investigate the gear transmission error including contact

elements, the whole bodies of gears have to be modeled because the penalty of parameter of the contact elements must account for the flexibility of the whole bodies of gears, not just the local stiffness.

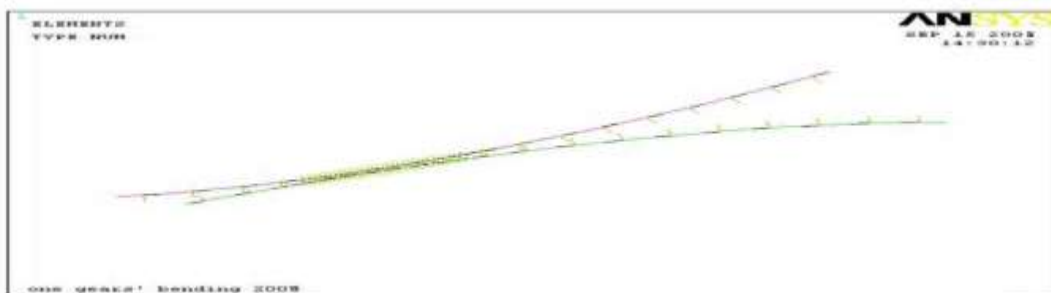


Figure 3.16 The beam elements were used in the FEA model

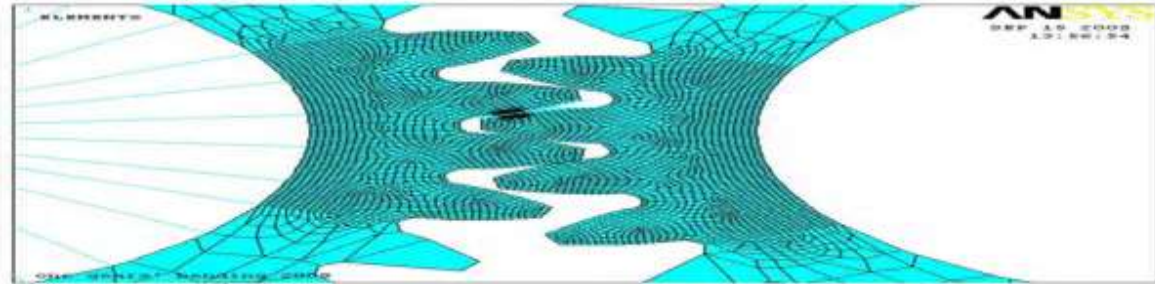


Figure 3-20 Meshing model for spur gears

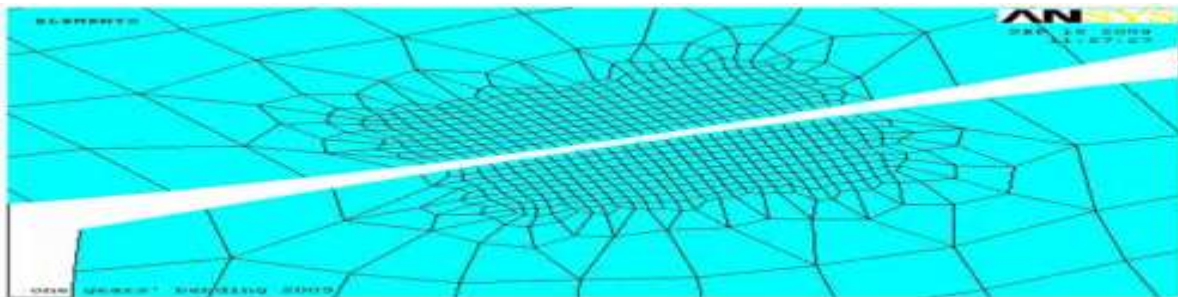


Figure 3-21 The fine mesh near the two contact surfaces

Here 2D plane42 elements were used with 2 degrees of freedom per node. The whole model has 5163 nodes, 4751 elements. For the contact surface the contact element was Conta172 and for the target surface the target element was Targe169 shown in Figure 3.19 that matches the position in Figure 3.20. Figure 3.20 displays a meshing model of a spur gear. Fine meshing was used shown in Figure 3.21. The one or two sets of contact elements were enlarged for the single or the double pairs of gears in contact. This operation allows extracting the

compliance due to bending and shear deformation, including the contact deformation. This procedure was successively applied to the pinion and the gear

### Overcoming the convergence difficulties

In this study the contact stress was always emphasized. The contact problem is usually a challenging problem because contact is a strong nonlinearity. Both the normal and tangential stiffness at the contact surfaces change significantly with changing contact status. Those kinds of large, sudden changes

in stiffness often cause severe convergence difficulties. If the constraints established on the model are not proper, it will result zero overall stiffness. In a static analysis unconstrained free bodies are

mathematically unstable and the solution “blows up”. In addition, the solution will not be in convergence if the total number of degrees of freedom exceeds 1,000,000.

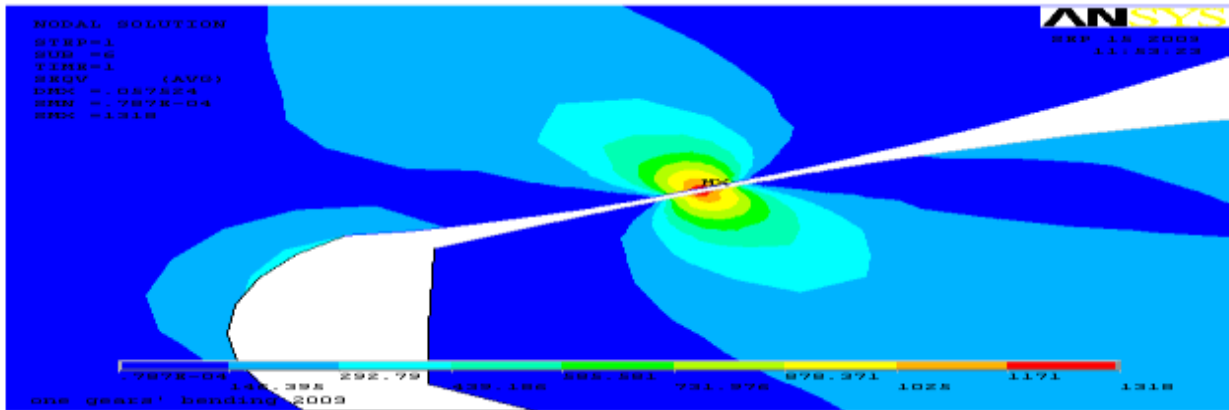


Figure 3-22 Von Mises stresses in spur gears

In this model, there are 4751 elements and 5163 nodes. For the contact surfaces there are more than eight nodes on each contact side. So the distribution of contact stresses is reasonable. In this chapter the transmission error is emphasized and contact is a nonlinear problem so the solution will likely

be done after a greater time compared with the time in linear analysis. It is much simpler to use “WIZARD BAR” and to create contact pair between the contact surface from “Preprocessor >Modeling >Create >Contact Pair”.

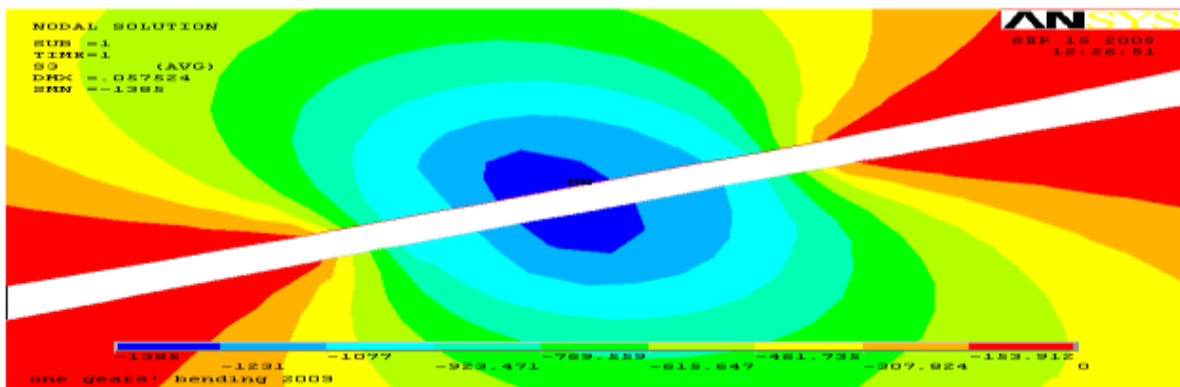


Figure 3-23 The distribution of contact stresses between two teeth

Here Von Mises stresses and the contact

stresses just for one position are shown

below in Figure 3.22 and Figure 3.23. For the gears the contact stress was compared with the results from the Hertz equations, and the two results agree with each other well.

### The Result of Transmission Error

This section considers a FEA model, which was used to predict static transmission error of a pair of spur gears in mesh including the contact deformation. The model involves the use of 2-D elements, coupled with contact elements near the points of contact between the meshing teeth. When one pair of teeth is meshing one set of contact elements was established between the two contact

surfaces, while when two pairs of teeth are meshing two sets of contact elements were established between the two contact bodies. When gears are unloaded, a pinion and gear with perfect involutes profiles should theoretically run with zero transmission error. However, when gears with involutes profiles are loaded, the individual torsional mesh stiffness of each gear changes throughout the mesh cycle, causing variations in angular rotation of the gear body and subsequent transmission error. The theoretical changes in the torsional mesh stiffness throughout the mesh cycle match the developed static transmission error using finite element analysis shown in Figure 4.4.

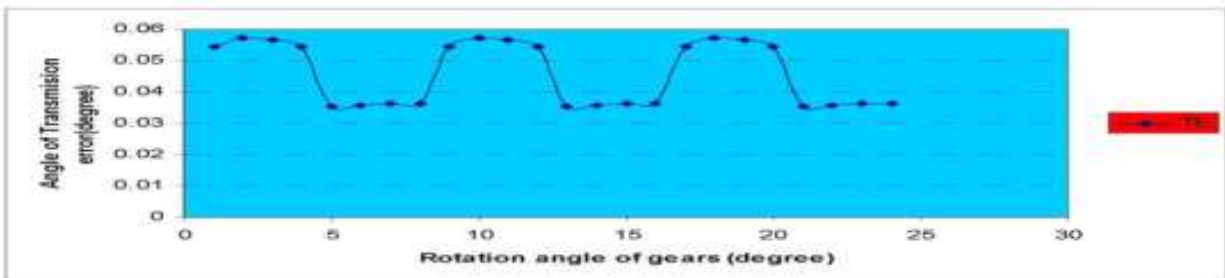


Figure 4.4 Static transmission errors from ANSYS

### Conclusions

- (i) Mesh stiffness variation as the number of teeth in contact changes is a primary cause of excitation of gear vibration and noise. This excitation exists even when the gears are perfectly machined and assembled. Numerical methods using 2-D FEM modeling of toothed bodies including contact elements have been developed to analyze the main static transmission error for spur gear pairs. Numerous simulations allowed validating this method and showed that a correct prediction of transmission error needed an accurate modeling of the whole toothed bodies. The

elasticity of those bodies modifies the contact between loaded tooth pairs and the transmission error variations. The developed numerical method allows one to optimize the static transmission error characteristics by introducing the suitable tooth modifications. These offer interesting possibilities as first steps of the development of a transmission system and can be also successfully used to improve to control the noise and vibration generated in the transmission system.

### REFERENCES



- 1) Tordian, G. V., 1967, "dynamic Measurement of the Transmission Error in Gear", Proc. JSME Semi-International Symposium September, pp. 279-287.
- 2) Sweeney, P. J., 1994, "Gear transmission error measurement and analysis", PhD dissertation, University of New South Wales, Australia.
- 3) Wang, J., 2003, "Survey of Nonlinear Vibration of Gear Transmission Systems" Appl. Mech. Rev. Vol 56, No 3.
- 4) Lim, T. C., 1991, "Vibration Transmission through Rolling Element Bearings, Part III: Geared Rotor System Studies", Journal of Sound and Vibration, 153(1), pp 31-54.
- 5) Perret-Liaudet, J., Sabot, J., 1994, "Dynamics of Truck Gearbox", 6th International Power Transmission and Gearing Conference, Scottsdale, Arizona, USA, pp 249-258.
- 6) Cockerham, G., 1967, "Computer-Aided Design of Spur or Helical Gear Train", Computer-Aided Design, Vol.8 No. 2, pp. 84-88.
- 7) Umezawa, K., 1988, "Recent Trends in Gearing Technology", JSME International Journal Series III Vol.31, No. 2, pp 357-362.
- 8) Sirichai, S. et al 1997, "Finite Element Analysis of gears in Mesh", Fifth International Congress on Sound and Vibration, Australia, pp 869-876.
- 9) Tavakoli M., Houser D., 1986, "Optimum Profile Modifications for the minimization of Static Transmission Errors of Spur Gears", Journal of Mechanisms, Transmission and Automation in Design, Vol. 108, pp 86-95.
- 10) Hamrock, B. J., Jacobson, S. R., "Fundamentals of Machine Elements".