ISSN (online): 2250-141X
Vol. 6 Issue 4, October 2016

A STATE OF THE ART REVIEW ON METHODS OF DETERMINATION OF ROBUST OPTIMUM PLOT SIZE

Satyabrata Pal[1],
Honorary Visiting Professor, Interdisciplinary Statistical Research Institute, Indian Statistical Institute, Kolkata 700108, India
Formerly, Dean Post Graduate Studies and Professor, Bidhan Chandra Krishi Viswavidyalaya, W.B., Pin-741252, India

Ayanendranath Basu
Professor (Higher Academic Grade), Interdisciplinary Statistical Research Institute, Indian Statistical Institute, Kolkata 700108, India

Goutam Mandal
Assistant Professor, Guru Nanak Institute of Technology, 157/F Nilganj Road, Sodepur, West Bengal – 700114, India .

Asim Kumar Chakraborty
Scientist, ICAR-CRIJAF, Nilganj, Barrackpore, Kolkata 700 120, West Bengal, India

Arunava Ghosh
Department of Agricultural Statistics, RRS, Terai Zone, Uttar Banga Krishi Viswavidyalaya, Pundibari, Cooch Behar-736165, West Bengal, India,

[1]Corresponding author

Abstract

The problem of determination of optimum size and shape of plots and blocks in field experiments has lured the statisticians and agriculturists for about a century. To trace its origin in the strictest sense of the term the celebrated works of Smith (1937) is to be recalled. The above problem has found its place as a full Chapter in the time-tested text book composed by Oscar Kempthorne in the fifties of the twentieth century. The aspect of soil heterogeneity governs the problem of determination of optimum plot sizes. Because of the diverse character of the aspect of soil heterogeneity existing in controlled field experiments, exact determination of optimum plot size has assumed a real challenge. Smith's formula has still been regarded as a fundamental step to understand the nature of soil heterogeneity in field experiment (based on uniformity trial data). Smith has worked out the formulae for determination of optimum plot size taking the relative cost components in cases of isotropy and anisotropy. Many papers are available in the literature to address the problem (determination of optimum plot size) with consideration of elements to fit into their situations described in their respective papers. An account on the determination of optimum shape and size of plots in the existing literature has been presented in Saste and Sananse (2015). None of available papers considers the underlying models (with specific mathematical expression defining soil heterogeneity in the background) which are supposed to govern the real-life data sets emanated from field experiments. This paper (intended to be a review) addresses the determination of robust optimum plot size and shape under model-based presentations imposed on

ISSN (online): 2250-141X
Vol. 6 Issue 4, October 2016

the field (uniformity trial) data to address the pattern of soil heterogeneity-structure prevailing often in field data as referred to in existing in literature till date. Two specific field heterogeneity structures have been considered and the robust optimum plot sizes are determined under the above two different situations. The robust optimum plot sizes are determined by employing the concept of variogram on correlated data (obtained from uniformity trial) under two different heterogeneity-situations. Though the technique of variogram is to be used on uniformity trial data, however, in cases of designed situations (CRD, RBD or LSD) the technique of variogram is to be applied on the residuals only to determine the robust optimum plot sizes. The papers on which this review is based are, Pal, et al (2007), Pal, et al (2015) and Pal and Basak (2015) respectively.

Keywords: soil heterogeneity structure**,** correlated data, variogram, radius of curvature, robust optimum plot sizes.

Introduction

Usually, the nature and extent of the fertility variation in land is attempted to be understood from analysing the data obtained from Uniformity trials being conducted on field. Uniformity trial is planned to determine the optimum size and shape of the plot. Uniformity trial involves growing a particular crop on a field or piece of land under uniform conditions, so as to say , in such a way that all sources of possible variation except that due to the native soil differences are kept at constant levels (more elaborately, the possible sources of variation as can arise from the application of manure, from application of irrigation level, from existence of the environment (i.e., temperature, sunlight, wind velocity, moisture content, etc.) prevailing at the place of experimentation, are maintained at fixed levels throughout the entire span of cultivation-process of the crop). At the time of harvest the entire field is divided into smallest units (say, 1m by 1m) of the same size and shape of plots. At this juncture, one may remember that the smallest (possible) is the size of the basic unit, the maximum detailed information can be extracted in respect of the measurement of the soil heterogeneity. In the past, many research workers have attempted to study the soil fertility variation and it is evidenced from their research that various other sources of variation (like, spread of pests and diseases) besides those already referred to are confounded with the soil, and it is also required to control such factors at the time of the conduction of the controlled experiment (uniformity trial). Saste and Sananse (2015) present an account on the determination of optimum shape and size of plots in the existing literature.

Since the middle of the twentieth century it has been realized that the phenomenon of independence of observations (in field experimental data) is not common. More specifically, the fact of non-compliance of the property of independence (in the field data), even after maintaining all the fundamental principles of design of experiments as propounded by Fisher in the conduct of the experiment, in the above-said data (as obtained from controlled field experiments), it becomes relevant to look into the aspect concerning the presence of correlation in the field data. Thus the problem of determination of robust optimum plot size has assumed an important role in the light of actual field data.

ISSN (online): 2250-141X
Vol. 6 Issue 4, October 2016

In the existing literature two distinct models (Model –I and Model – II) are considered and this paper reviews the results on the determination of optimum plot sizes under the above said models. The above two cases of most commonly employed models (incorporating specified correlation structure in each case) assumed to represent the real-life data situations (uniformity trial or designed experiments) precisely have been considered.

Section 2 describes the analytical details when Model I is employed on the uniformity trial (UT) data and Section 3 is devoted to consideration of Model II on UT data

The robust optimum plot sizes are determined as, 2x5, 2x6, 2x7, 3x5, 3x6, 3x7, 4x5, 4x6, 4x7, and 5x5 respectively under the Model-I and under the Model-II, the robust optimum plot sizes are, 2x5, 2x6, 3x5 and 3x6 respectively.

Derivation under Model I

In real-life data from field experiments, existence of serial correlation is quite prevalent in published data sets in the literature namely, Smith (1938), Modjeska and Rawlings (1983), Webster and Burgess (1984), Sethi (1985), Zhang et al.(1990, 1994), Bhatti et al.(1991), Fagroud and Meirvenne (2002), etc. In this paper a model which represents the real-life field situation obeying a special type of heterogeneity structure, has been considered.

Let Y (**s**) $\{\mathbf{s} \in D_s \subset R^2$; Y (**s**) values being 1-dimensional $\}$ be a real valued spatial process defined on a domain D_s of the 2-dimensional Euclidean space R^2, and it is supposed that the variance of the difference of the values, (Y (**s**)) of the variable (Y) at two, two-dimensional locations, $\mathbf{s_1} = (h_{10}, h_{20})$ and $\mathbf{s_2} = (h_{11}, h_{21})$ (displaced **h**-apart, $\mathbf{h} = \mathbf{s_2} - \mathbf{s_1}$**,),** depends on **h**. More specifically, it is assumed that the variogram, defined as, $Var [Y(\mathbf{s_2}) - Y(\mathbf{s_1})] = 2\gamma(\mathbf{h})$, for all $(\mathbf{s_1}, \mathbf{s_2}) \in D_s$, the variogram must satisfy the conditional-non-positive-definiteness condition. The function, $\gamma(\mathbf{h})$ is called the semi-variogram. The quantity $2\gamma(\mathbf{h})$ being a function of the difference between the spatial locations, $s_1 = \mathbf{s}$ and $s_2 = \mathbf{s} + \mathbf{h}$**,** is called the stationary variogram. When $2\gamma(\mathbf{h})$ becomes independent of s, and is a function of $\|\mathbf{h}\|$ only, for $\mathbf{h} = (h_1, h_2) \in R^2$, $\|h\| = (h_1^2 + h_2^2)^{1/2}$, the variogram is said to be isotropic, otherwise, it is said to be anisotropic. Relevant literature on the concept of variogram the following references may be consulted: Matheron (1963), Cressie (1993) and Cressie and Wikle (2011).

In this paper, a special real-life data situation, where the model-structure (under uniformity trial data, Y (**s**) on a spatial location **s**) is extensively investigated and its model expression is presented as follows:

Model I : $Y(\mathbf{s}) = \mu + e(\mathbf{s})$, $V(Y(\mathbf{s})) = V(e(\mathbf{s})) = \sigma^2$; $Cov(Y(\mathbf{s}), Y(\mathbf{s}+\mathbf{h})) = Cov(e(\mathbf{s}), e(\mathbf{s}+\mathbf{h})) = \rho^{|h_1|+|h_2|}\sigma^2$

Ample applications of Model I are available in literature (for representing corresponding spatial situation). Model (1) in unidimensional case find its prime inclusion in literature in the early fifties of the twentieth century in the paper by Williams (1952), and subsequently by many others including Bartlett (1978).

Variogram of the **residuals** are to be modelled in case of data from designed experiments (RBD, LSD, incomplete blocks etc.) adopting the method delineated in the paper.

ISSN (online): 2250-141X
Vol. 6 Issue 4, October 2016

In order to select optimum plot sizes, the criteria, mentioned in section 2 are used.

The alternative robust optimum plot sizes are determined as, 2x5, 2x6, 2x7, 3x5, 3x6, 3x7, 4x5, 4x6, 4x7 and 5x5, respectively under the above Model-I, the values of the intra-class correlation (ρ) of the first order being taken as: ρ = 0.1, ρ = 0.2, ρ = 0.3, ρ = 0.4, ρ = 0.5, respectively, in fact, higher values of ρ are not evidenced in real-life data from field experiments. In order to take care of spatial heterogeneity in two directions (row and column), the choices in regard to the plot sizes have been restricted to plot sizes, 2 x 2, and higher.

The expressions of the theoretical variograms, 2 γ(**h**) (under the Model I) have been obtained for plot sizes, l x k (l = 2, 3, ……; k = 2, 3, ………), i.e., 2 x 2, 2 x 3, 2 x 4, 2 x 5, 2 x 6, 2 x 7, 2 x 8, 3 x 3, 3 x 4, 3 x 5, 3 X 6, 3 x 7, 3 x 8, 4 x 4, 4 x 5, 4 x 6, 4 x 7, 5 x 5, etc, respectively (area of plots being less than or equal to 30 squared units are explored) and a few of them are presented in the Table 1.

Table 1

Expressions of the theoretical variograms (a few only) in case of the plot sizes

Plot size (l x k)	2 γ (**h**)
1 x 1	$\sigma^2(2-2\rho^h)$
2 x 2	$\sigma^2[8+4(2\rho^2+4\rho)-2(2\rho^{2h+2}+6\rho^{2h+1}+6\rho^{2h}+2\rho^{2h-1})]$
2 x 5	$\sigma^2[20+4(2\rho^5+6\rho^4+10\rho^3+14\rho^2+13\rho)-2(2\rho^{5h+5}+6\rho^{5h+4}+10\rho^{5h+3}+14\rho^{5h+2}+18\rho^{5h+1}+18\rho^{5h}+14\rho^{5h-1}+10\rho^{5h-2}+6\rho^{5h-3}+2\rho^{5h-4})]$
2 x 8	$\sigma^2[32+4(2\rho^8+6\rho^7+10\rho^6+14\rho^5+18\rho^4+22\rho^3+26\rho^2+22\rho)-2(2\rho^{8h+8}+6\rho^{8h+7}+10\rho^{8h+6}+14\rho^{8h+5}+18\rho^{8h+4}+22\rho^{8h+3}+26\rho^{8h+2}+30\rho^{8h+1}+30\rho^{8h}+26\rho^{8h-1}+22\rho^{8h-2}+18\rho^{8h-3}+14\rho^{8h-4}+10\rho^{8h-5}+6\rho^{8h-6}+2\rho^{8h-7})]$
3 x 3	$\sigma^2[18+4(2\rho^4+8\rho^3+14\rho^2+12\rho)-2(2\rho^{3h+4}+8\rho^{3h+3}+17\rho^{3h+2}$

ISSN (online): 2250-141X
Vol. 6 Issue 4, October 2016

	$+22\rho^{3h+1}+19\rho^{3h}+10\rho^{3h-1}+3\rho^{3h-2})]$
3 x 4	$\sigma^2[24+4(2\rho^5+8\rho^4+17\rho^3+20\rho^2+17\rho)-2(2\rho^{4h+5}+8\rho^{4h+4}$ $+17\rho^{4h+3}+26\rho^{4h+2}+31\rho^{4h+1}+28\rho^{4h}+19\rho^{4h-1}+10\rho^{4h-2}$ $+3\rho^{4h-3})]$
3 x 6	$\sigma^2[36+4(2\rho^7+8\rho^6+17\rho^5+26\rho^4+35\rho^3+38\rho^2+27\rho)-2(2\rho^{6h+7}$ $+8\rho^{6h+6}+17\rho^{6h+5}+26\rho^{6h+4}+35\rho^{6h+3}+44\rho^{6h+2}+49\rho^{6h+1}$ $+46\rho^{6h}+37\rho^{6h-1}+28\rho^{6h-2}+19\rho^{6h-3}+10\rho^{6h-4}+3\rho^{6h-5})]$
3 x 7	$\sigma^2[42+4(2\rho^8+8\rho^7+17\rho^6+26\rho^5+35\rho^4+44\rho^3+46\rho^2+32\rho)$ $-2(2\rho^{7h+8}+8\rho^{7h+7}+17\rho^{7h+6}+26\rho^{7h+5}+35\rho^{7h+4}+44\rho^{7h+3}$ $+53\rho^{7h+2}+58\rho^{7h+1}+55\rho^{7h}+46\rho^{7h-1}+37\rho^{7h-2}+28\rho^{7h-3}$ $+19\rho^{7h-4}+10\rho^{7h-5}+3\rho^{7h-6})]$
3 x 8	$\sigma^2[48+4(2\rho^9+8\rho^8+17\rho^7+26\rho^6+35\rho^5+44\rho^4+53\rho^3+54\rho^2+37\rho)$ $-2(2\rho^{8h+9}+8\rho^{8h+8}+17\rho^{8h+7}+26\rho^{8h+6}+35\rho^{8h+5}+44\rho^{8h+4}$ $+53\rho^{8h+3}+62\rho^{8h+2}+67\rho^{8h+1}+64\rho^{8h}+55\rho^{8h-1}+46\rho^{8h-2}$ $+37\rho^{8h-3}+28\rho^{8h-4}+19\rho^{8h-5}+10\rho^{8h-6}+3\rho^{8h-7})]$

*

4 x 4	$\sigma^2[32+4(2\rho^6+8\rho^5+20\rho^4+32\rho^3+32\rho^2+24\rho)-2(2\rho^{4h+6}$ $+8\rho^{4h+5}+20\rho^{4h+4}+36\rho^{4h+3}+48\rho^{4h+2}+52\rho^{4h+1}+44\rho^{4h}$ $+28\rho^{4h-1}+14\rho^{4h-2}+4\rho^{4h-3})]$

ISSN (online): 2250-141X
Vol. 6 Issue 4, October 2016

4 x 5	$\sigma^2[40+4(2\rho^7+8\rho^6+20\rho^5+36\rho^4+47\rho^3+46\rho^2+31\rho)-2(2\rho^{5h+7}$ $+8\rho^{5h+6}+20\rho^{5h+5}+36\rho^{5h+4}+52\rho^{5h+3}+64\rho^{5h+2}+68\rho^{5h+1}$ $+60\rho^{5h}+44\rho^{5h-1}+28^{5h-2}+14\rho^{5h-3}+4\rho^{5h-4})]$
4 x 8	$\sigma^2[64+4(2\rho^{10}+8\rho^9+20\rho^8+36\rho^7+52\rho^6+68\rho^5+84\rho^4+92\rho^3$ $+82\rho^2+52\rho)-2(2\rho^{8h+10}+8\rho^{8h+9}+20\rho^{8h+8}+36\rho^{8h+7}$ $+52\rho^{8h+6}+68\rho^{8h+5}+84\rho^{8h+4}+100\rho^{8h+3}+112\rho^{8h+2}$ $+116\rho^{8h+1}+108\rho^{8h}+92\rho^{8h-1}+76\rho^{8h-2}+60\rho^{8h-3}$ $+44\rho^{8h-4}+28\rho^{8h-5}+14\rho^{8h-6}+4\rho^{8h-7})]$

*

5 x 5	$\sigma^2[50+4(2\rho^8+8\rho^7+20\rho^6+40\rho^5+60\rho^4+68\rho^3+62\rho^2+40\rho)$ $-2(2\rho^{5h+8}+8\rho^{5h+7}+20\rho^{5h+6}+40\rho^{5h+5}+65\rho^{5h+4}+86\rho^{5h+3}$ $+99\rho^{5h+2}+100\rho^{5h+1}+85\rho^{5h}+60\rho^{5h-1}+37^{5h-2}+18\rho^{5h-3}$ $+5\rho^{5h-4})]$
5 x 6	$\sigma^2[60+4(2\rho^9+8\rho^8+20\rho^7+40\rho^6+65\rho^5+84\rho^4+89\rho^3+78\rho^2$ $+49\rho)-2(2\rho^{6h+9}+8\rho^{6h+8}+20\rho^{6h+7}+40\rho^{6h+6}+65\rho^{6h+5}$ $+90\rho^{6h+4}+111\rho^{6h+3}+124\rho^{6h+2}+125\rho^{6h+1}+110\rho^{6h}$ $+85\rho^{6h-1}+60\rho^{6h-2}+37\rho^{6h-3}+18\rho^{6h-4}+5\rho^{6h-5})]$

*

[The theoretical expression for 2 γ (**h**) for each plot size becomes a function of the pair of parameters (“h” and “ρ”). In order to select the optimum plot size, the value of h_{opt} is

ISSN (online): 2250-141X
Vol. 6 Issue 4, October 2016

determined with respect to a particular plot size, h_{opt} being the point in the domain of "h" at which the value of the radius of curvature (with respect to the variogram curve, 2 ɣ (h) plotted against "h"), is minimum, at fixed value of "ρ" from 0.1 to 0.5 successively (for very close values of the radius of curvature with respect to a few plot sizes, optimum plot size may not be uniquely determined). In case, the same set of optimum plot sizes exist against different values of "ρ" as mentioned earlier, the set of optimum plot sizes becomes "robust". The formula for radius of curvature ("r_c") of the variogram curve is given below:

$$r_c = (1 + \gamma_1(h)^2)^{3/2} / (\gamma_2(h)), \text{ where } \gamma_1(h) = d\gamma/dh \text{ and } \gamma_2(h) = d^2\gamma/dh^2.$$

Some other restrictions are to be applied in the process of determination of robust optimum plot sizes (conditions include, the difference between the two dimensions, length and breadth, for any plot size can't exceed 5, square plots are not advisable, plots with unit dimension on any side are not acceptable.

For values of ρ = 0.1 to ρ = 0.5, the alternative optimum plot sizes are determined as, 2x5, 2x6, 2x7, 3x5, 3x6, 3x7, 4x5, 4x6 and 4x7 irrespective of the value of ρ.

Thus Robust (for ρ=0.1 to ρ=0.5) optimum plot sizes (in squared units) are: 10/12/14/15/18/21/20/24/28.

Derivation under Model II

Recalling the Uniformity trial data Y (s) on a spatial location s is modeled as:
$Y(s) = \mu + e(s)$, $V(Y(s)) = V(e(s)) = \sigma^2$; $Cov(Y(s), Y(s+h)) = Cov(e(s), e(s+h)) = \rho^{||h||}\sigma^2$, $||h|| = (h_1^2 + h_2^2)^{1/2}$. In case of data from the designed experiments, variogram of the residuals are to be modeled in the above manner.

As in case of the Model I, the expressions of the theoretical variograms, 2 γ (h) (under Model II) have been obtained for plot sizes, l x k (l = 2, 3, ……; k = 2, 3, ………), i.e., 2 x 2, 2 x 3 (3 x 2), 2 x 4 (4 X 2), 2 x 5 (5 x 2), 2 x 6 (6 X 2), 2 x 7 (7 x 2), 2 x 8 (8 X 2), 3 x 3, 3 x 4 (4 X 3), 3 x 5 (5 X 3), 4x4, etc, respectively (area of plots being less than or equal to 16 squared units), and some of such expressions have been presented as follows.

Expressions of variograms (for some plot sizes only out of the above plot sizes) are given as follows:

ISSN (online): 2250-141X
Vol. 6 Issue 4, October 2016

1 x 1: $2\gamma(h) = 2\sigma^2 (1-\rho^h)$

2 x 2:

$$2\gamma(h) = 8\sigma^2 + 2\sigma^2[(8\rho + 4\rho^{\sqrt{2}}) - (2\rho^{(2h-1)} + 4\rho^{(2h)} + 2\rho^{(2h+1)} + 2\rho^{\sqrt{(2h-1)^2+1}} + 4\rho^{\sqrt{(2h)^2+1}} + 2\rho^{\sqrt{(2h+1)^2+1}})]$$

2 x 3:

$$2\gamma(h) = 12\sigma^2 + 2\sigma^2[(14\rho + 4\rho^2 + 8\rho^{\sqrt{2}} + 4\rho^{\sqrt{5}}) - (2\rho^{(3h-2)} + 4\rho^{(3h-1)} + 6\rho^{(3h)} + 4\rho^{(3h+1)} + 2\rho^{(3h+2)} + 2\rho^{\sqrt{(3h-2)^2+1}} + 4\rho^{\sqrt{(3h-1)^2+1}} + 6\rho^{\sqrt{(3h)^2+1}} + 4\rho^{\sqrt{(3h+1)^2+1}} + 2\rho^{\sqrt{(3h+2)^2+1}})]$$

2 x 5:

$$2\gamma(h) = 20\sigma^2 + 2\sigma^2[(26\rho + 12\rho^2 + 8\rho^3 + 4\rho^4 + 16\rho^{\sqrt{2}} + 12\rho^{\sqrt{5}} + 8\rho^{\sqrt{10}} + 4\rho^{\sqrt{17}}) - (2\rho^{(5h-4)} + 4\rho^{(5h-3)} + 6\rho^{(5h-2)} + 8\rho^{(5h-1)} + 10\rho^{(5h)} + 8\rho^{(5h+1)} + 6\rho^{(5h+2)} + 4\rho^{(5h+3)} + 2\rho^{(5h+4)} + 2\rho^{\sqrt{(5h-4)^2+1}} + 4\rho^{\sqrt{(5h-3)^2+1}} + 6\rho^{\sqrt{(5h-2)^2+1}} + 8\rho^{\sqrt{(5h-1)^2+1}} + 10\rho^{\sqrt{(5h)^2+1}} + 8\rho^{\sqrt{(5h+1)^2+1}} + 6\rho^{\sqrt{(5h+2)^2+1}} + 4\rho^{\sqrt{(5h+3)^2+1}} + 2\rho^{\sqrt{(5h+4)^2+1}})]$$

3 x 3:

$$2\gamma(h) = 18\sigma^2 + 2\sigma^2[(246\rho + 12\rho^2 + 16\rho^{\sqrt{2}} + 16\rho^{\sqrt{5}} + 4\rho^{\sqrt{8}}) - (3\rho^{(3h-2)} + 6\rho^{(3h-1)} + 9\rho^{(3h)} + 6\rho^{(3h+1)} + 3\rho^{(3h+2)} + 4\rho^{\sqrt{(3h-2)^2+1}} + 2\rho^{\sqrt{(3h-2)^2+4}} + 8\rho^{\sqrt{(3h-1)^2+1}} + 4\rho^{\sqrt{(3h-1)^2+4}} + 12\rho^{\sqrt{(3h)^2+1}} + 6\rho^{\sqrt{(3h)^2+4}} \; 8\rho^{\sqrt{(3h+1)^2+1}} + 4\rho^{\sqrt{(3h+1)^2+4}} \; 4\rho^{\sqrt{(3h+2)^2+1}} + 2\rho^{\sqrt{(3h+2)^2+4}})]$$

3 x 5:

$$2\gamma(h) = 30\sigma^2 + 2\sigma^2[(44\rho + 28\rho^2 + 12\rho^3 + 6\rho^4 + 32\rho^{\sqrt{2}} + 40\rho^{\sqrt{5}} + 12\rho^{\sqrt{8}} + 16\rho^{\sqrt{10}} + 8\rho^{\sqrt{17}}) - (3\rho^{(5h-4)} + 6\rho^{(5h-3)} + 9\rho^{(5h-2)} + 12\rho^{(5h-1)} + 15\rho^{(5h)} + 12\rho^{(5h+1)} + 9\rho^{(5h+2)} + 6\rho^{(5h+3)} + 3\rho^{(5h+4)} + 4\rho^{\sqrt{(5h-4)^2+1}} + 2\rho^{\sqrt{(5h-4)^2+4}} + 8\rho^{\sqrt{(5h-3)^2+1}} + 4\rho^{\sqrt{(5h-3)^2+4}} + 12\rho^{\sqrt{(5h-2)^2+1}} + 6\rho^{\sqrt{(5h-2)^2+4}} + 16\rho^{\sqrt{(5h-1)^2+1}} + 8\rho^{\sqrt{(5h-1)^2+4}} + 20\rho^{\sqrt{(5h)^2+1}} + 10\rho^{\sqrt{(5h)^2+4}} + 16\rho^{\sqrt{(5h+1)^2+1}} + 8\rho^{\sqrt{(5h+1)^2+4}} + 12\rho^{\sqrt{(5h+2)^2+1}} + 6\rho^{\sqrt{(5h+2)^2+4}} + 8\rho^{\sqrt{(5h+3)^2+1}} + 4\rho^{\sqrt{(5h+3)^2+4}} + 4\rho^{\sqrt{(5h+4)^2+1}} + 2\rho^{\sqrt{(5h+4)^2+4}})]$$

ISSN (online): 2250-141X
Vol. 6 Issue 4, October 2016

4 x 4:

$$
\begin{aligned}
2\gamma(h) = {} & 32\sigma^2 + 2\sigma^2[(48\rho + 32\rho^2 + 16\rho^3 + 36\rho^{\sqrt{2}} + 48\rho^{\sqrt{5}} + 16\rho^{\sqrt{8}} + 24\rho^{\sqrt{10}} + 16\rho^{\sqrt{13}}) \\
& - (4\rho^{(4h-3)} + 8\rho^{(4h-2)} + 12\rho^{(4h-1)} + 16\rho^{(4h)} + 12\rho^{(4h+1)} + 8\rho^{(4h+2)} + 4\rho^{(4h+3)} \\
& + 6\rho^{\sqrt{(4h-3)^2+1}} + 4\rho^{\sqrt{(4h-3)^2+4}} + 2\rho^{\sqrt{(4h-3)^2+9}} + 12\rho^{\sqrt{(4h-2)^2+1}} + 8\rho^{\sqrt{(4h-2)^2+4}} + 4\rho^{\sqrt{(4h-2)^2+9}} \\
& + 18\rho^{\sqrt{(4h-1)^2+1}} + 12\rho^{\sqrt{(4h-1)^2+4}} + 6\rho^{\sqrt{(4h-1)^2+9}} + 24\rho^{\sqrt{(4h)^2+1}} + 16\rho^{\sqrt{(4h)^2+4}} + 8\rho^{\sqrt{(4h)^2+9}} \\
& + 18\rho^{\sqrt{(4h+1)^2+1}} + 12\rho^{\sqrt{(4h+1)^2+4}} + 6\rho^{\sqrt{(4h+1)^2+9}} + 12\rho^{\sqrt{(4h+2)^2+1}} + 8\rho^{\sqrt{(4h+2)^2+4}} + 4\rho^{\sqrt{(4h+2)^2+9}} \\
& + 6\rho^{\sqrt{(4h+3)^2+1}} + 4\rho^{\sqrt{(4h+3)^2+4}} + 2\rho^{\sqrt{(4h+3)^2+9}})]
\end{aligned}
$$

The selection of optimum (best or better) plot sizes is governed by the following criteria:

1. With respect to each plot size the point h_{opt} is determined (for which point (i.e., h_{opt}) the value of the radius of curvature, r_c, is minimum), the formula of radius of curvature is given below:

$$r_c = (1 + \gamma_1(h)^2)^{3/2} / (\gamma_2(h)), \text{ where } \gamma_1(h) = d\gamma/dh \text{ and } \gamma_2(h) = d^2\gamma/dh^2.$$

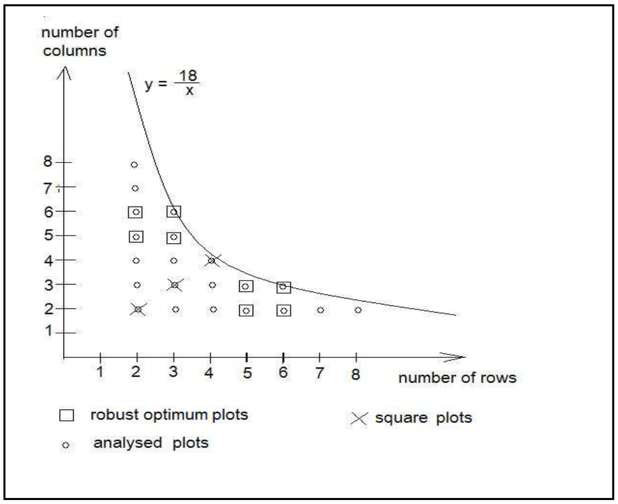
2. For each value of ρ, say ρ = .1, the particular plot size is chosen for which the values of the radius of curvature, r_c, are minimum/(near to minimum) with the restriction that $|l - k| \leq 4$.

3. Plots with unit dimension in any direction (row or column) are not taken into account, also long narrow plots are not recommended (owing to the fact that such plots imbibe more heterogeneity).

Square plots of sizes, 2x2, 3x3, 4x4 possess values of radii of curvature much higher than the desired minimum values (meaning there by the curve shapes corresponding to those plot sizes are relatively more flat), thus square plots can't be taken as optimum plot sizes. Though the radii of curvature corresponding to plots of sizes, 2x7 and 2x8 are less than the minimum values of r_c's taken into consideration, yet such plot sizes are not recommended as optimum plot sizes, as these plots are of long and narrow shape. Plots sizes, 5x5 and 6x6 are not taken into account as such plots are of big sizes.

For ρ = 0.1 to ρ = 0.5, the robust optimum plot sizes are: 2x5, 2x6, 3x5 and 3x6 respectively.

In the following Graph the optimum plots are shown.



ISSN (online): 2250-141X
Vol. 6 Issue 4, October 2016

References

I. Bartlett, M.S. 1978. Nearest neighbor models in the analysis of field experiment (with discussion). J.R. Statist. Soc. B 40, 147-74.

II. Bhatti A.U., Mulla D.J., Koehler F.E., Gurmani A.H. 1991. Identifying and removing spatial correlation from yield experiments. Soil Sci. Soc. Am. J. 55,1523-1528.

III. Cressie N.A.C. 1993, Statistics for Spatial Data. John Wiley, New York.

IV. Cressie N., Hawkins D.M. 1980. Robust estimation of the variogram, I. Jour. of International Association for Mathematical geology 12, 115-125.

V. Cressie N., Wikle C.K. 2011. Statistics for Spatio-Temporal Data. Pub. A John Wiley & Sons. Inc.

VI. Faground M., Meirvenne M. Van. 2002. Accounting for Soil Spatial Autocorrelation in the design of experimental trials. Soil Sci. Soc. Am. J. 66,1134-1142.

VII. Matheron G. 1963. Principles of geostatistics. Economic geology 58,1246-1266.

VIII. Modjeska J.S., Rawlings J.O. 1983. Spatial correlation analysis of uniformity data. Biometrics 39, 373-384.

IX. Pal S., Basak S., Pal S., Kageyama S. 2007: On determination of optimum size and shape of plots in field trials. Biometrical Letters, 44(1), 23-31.

X. Pal S., Maldal G., Dihidar K. 2015: Determination of robust optimum plot size and shape-a model-based approach, Biometrical Letters, 52(1), 13-22.

XI. Pal S., Basak S. 2016: Optimum Plot Size-Robust Determination, Asian Academic Research Journal of Multidisciplinary, 3(1), 278-290.

XII. Saste S.V, Sananse S.L. 2015. Soil Heterogeneity to Determine Size and Shape of Plots: A Review, International Journal of Advanced Scientific and Technical Research, 6(5), 201-209.

XIII. Sethi A.S. 1985. A modified approach to determine the optimum size and shape of plots in field experiments on maize grown on terraced land. Indian Jour. of Agril. Sci. 55 (1), 48-51.

XIV. Smith H.F. 1938. An empirical law describing heterogeneity in the yields of agricultural crops. Jour. of Agricultural Science, Cambridge 28, 1-29.

XV. Webster R., Burgess T.M. 1984. Sampling and bulking strategies for estimating soil properties in small regions. J. Soil Sci. 35, 127-140.

XVI. Zhang R, Warrick A.W., Myers D.E. 1990. Variance as a function of sample support size. Math. Geol. 22, no.1, 107-121.

XVII. Zhang R, Warrick A.W., Myers D.E. 1994. Heterogeneity, plot shape effect and optimum plot size. Geoderma, 62, 183-197.